



Rossmoyne Senior High School

Semester One Examination, 2024

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

WA student number: In figures

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Circle your teacher:

Ms Alvaro Ms Lee
Mr Liu Ms Robinson

In words

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

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Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	93	65
Total					100

Instructions to candidates

- The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	5	
2	9	
3	7	
4	8	
5	9	
6	7	
7	7	
Section 1 Total	52	
Weighted ($\times 0.7143$)	35%	
Section 2 Weighted	65%	
Total	100%	

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Section One: Calculator-free

35% (52 Marks)

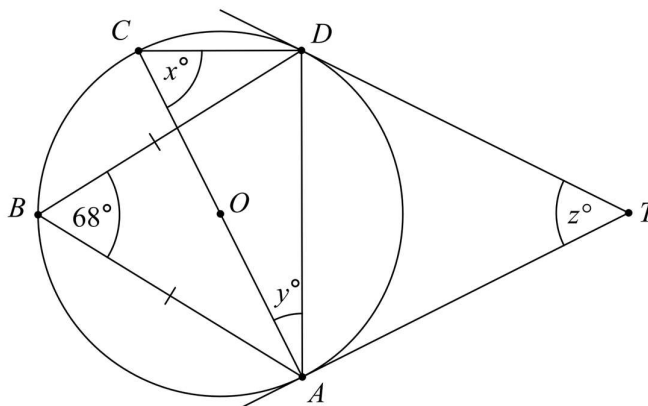
This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1

(5 marks)

In the diagram below, $BD = BA$, $\angle DBA = 68^\circ$ and DT and AT are tangents to the circle centred at O .



Determine, giving reasons, the value of:

(a) x (1 mark)

(b) y (2 marks)

(c) z (2 marks)

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Question 2

(9 marks)

- (a) A student wants to prove that the following statement is true for all integers, x :

If x^2 is even, then x is even

To prove that this statement is true, it is sufficient to prove which of the following:

- A** If x^2 is odd, then x is odd **B** If x is odd, then x^2 is odd
C If x is even, then x^2 is even **D** If x is odd, then x^2 is even

Explain how you made your choice. You are **not** required to prove it is true. (3 marks)

- (b) A set contains n distinct positive integers. Prove that at least two integers in this set have the same remainder when divided by $n - 1$. (3 marks)

- (c) Determine the truth of the following statement, where m , n and p are integers. Explain how you made your decision. (3 marks)

$$\forall n \forall m \exists p: p = \frac{m+n}{2}$$

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Question 3

(7 marks)

- (a) Consider any point R that lies on the circumference of a circle with centre O and diameter PQ . Using only triangle properties, prove that $\angle PRQ = 90^\circ$. (3 marks)

- (b) Points A, B, C, D and E lie on circle with centre O and diameter AD as shown in the diagram to the right (not to scale).

$$\angle BAC = 25^\circ.$$

$$\angle BCA = 18^\circ.$$

$$\angle ACE = 54^\circ.$$

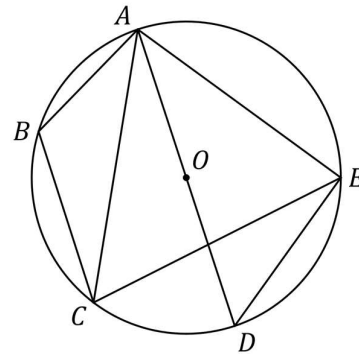
Determine the size of the following angles:

(i) $\angle OAE$.

(2 marks)

(ii) $\angle CED$.

(2 marks)



Question 4

(8 marks)

- (a) State, with justification, whether the vectors $\begin{pmatrix} 4 \\ 24 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$ are parallel, perpendicular or neither. (1 mark)
- (b) Determine the vector projection of $\begin{pmatrix} 2 \\ 14 \end{pmatrix}$ on $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. (2 marks)
- (c) Determine the value(s) of the constant μ given that vectors $\begin{pmatrix} 6 \\ \mu \end{pmatrix}$ and $\begin{pmatrix} -2 \\ \mu - 1 \end{pmatrix}$ are perpendicular. (2 marks)
- (d) Evaluate $(\underline{\underline{r}} - 4\underline{\underline{s}}) \cdot (3\underline{\underline{s}} + 2\underline{\underline{r}})$ given that $\underline{\underline{r}} \cdot \underline{\underline{s}} = -10$ and vectors $\underline{\underline{r}}$ and $\underline{\underline{s}}$ have magnitudes 4 and 5 respectively. (3 marks)

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Question 5

(9 marks)

Trolleys at a shopping centre are collected from a car park using a trailer. The trailer can hold a maximum of five trolleys.

Trolleys can come from any of four stores: Aldo, Coldo, The Project Shop or W-Mart. Each store's trolleys are different from each other, but all trolleys from the same store are identical.

- (a) How many ways can five trollies be put on the trailer if
- (i) two Coldo shopping trolleys have been rammed together and cannot be separated, and there is one trolley from each of the other three stores? (1 mark)
 - (ii) there are three trolleys from The Project Shop and two from W-Mart? (2 marks)
 - (iii) if there is at least one trolley from each store? (3 marks)

People need to pay to use the Aldo trolleys. To get a refund, each Aldo trolley needs to be chained to another Aldo trolley. Hence, Aldo trolleys are often chained together.

- (b) How many ways can five trollies be put on the trailer if there are **at least two** consecutive Aldo trolleys? (3 marks)

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Question 6

(7 marks)

Points A and B have position vectors $\vec{a} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

(a) Determine the vector $\overrightarrow{AB}$.

(1 mark)

(b) Determine the vectors $\vec{r}$ and $\vec{s}$ given that $\vec{a} = \vec{r} + 2\vec{s}$ and $\vec{b} = 2\vec{r} - \vec{s}$.

(3 marks)

(c) Determine the value(s) of the constant μ for which $|\vec{b} - \mu\vec{a}| = \sqrt{13}$.

(3 marks)

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Question 7

(7 marks)

- (a) A multiple-choice test consists of 4 questions, each with 5 possible answers A, B, C, D and E . Determine the least number of students who must take the test to ensure that there will be at least 3 students with identical sets of answers. Justify your answer. (3 marks)

- (b) Consider the identity from Pascal's triangle $5 \binom{n}{5} = (n-4) \binom{n}{4}$.

- (i) Show that the identity is true when $n = 5$. (1 mark)

- (ii) Use the definition $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ to prove the identity for $n \in \mathbb{Z}, n \geq 5$. (3 marks)

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Supplementary page

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